Solving Mathematical Word Problems with LEP Students

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Language minority students, particularly at the pre-university level, are often placed in mainstream classes with neither the language proficiency nor the academic skills necessary for functioning successfully. Due to the lack of adequate programs, these students seem destined for academic failure. Their inability to understand English limits their participation in academic activities and inhibits their success in school. Once placed in a class in which they cannot learn the material, they fall farther and farther behind as the years progress.

In order to avoid this inadequacy, schools are now adopting sheltered English programs, in which Limited English Proficient (LEP) students receive the same content as in the mainstream classes, but with additional emphasis on language development as well as language modification by the teacher. In this way, LEP students are able to advance their academic skills at the same rate as their English proficient peers. When the student achieves adequate language skills to be mainstreamed, her cognitive abilities will be sufficiently developed, that is, at a level which enables her to function successfully in the mainstream classroom.

At the early stages of language development, students can be placed in mainstream classes which are linguistically less-demanding such as art, music, or P.E. At later stages they are placed in more challenging classes such as science and math. Generally, however, math is thought of as one of the easier subjects for students from other countries, since it is often the case that they have studied more advanced mathematics in their home countries than what is expected of them in American schools at their particular levels. An added help is that a considerable amount of math requires limited, albeit specialized, language. However, mathematics is not limited to arithmetic. The National Council of Teachers of Mathematics (NCTM) states that math also involves problem solving and interpersonal communication, making reasoning skills crucial (cited in Secada and Carey 1990:10). The NCTM has outlined certain goals for all students of mathematics:

1. that they learn to value mathematics
2. that they become confident in their ability to do mathematics
3. that they become mathematical problem solvers
4. that they learn to communicate mathematically
5. that they learn to reason mathematically. (cited in Secada and Carey, 1990:9)

Perhaps the area which proves most difficult for LEP math students is word problems, sometimes called story problems. In word problems, language plays a primary role. Students must have a very good understanding of the language of the problem in order to integrate their conceptual knowledge with their cognitive skills. Unfortunately, the language of word problems is often much too difficult for LEP students, rendering them unable to solve the problem even though they may have the mathematical knowledge to do so.

> Mathematical problems cannot be solved without some concept of the basic facts, competence in computation, understanding of operations, or the ability to sequence facts in logical order. (Ferguson and Fairburn 1985:504)

If even one of these skills is lacking, students will not be successful in their attempts to solve word problems. When this happens, the students are effectively prevented from furthering their logic and reasoning skills. For LEP students, whose symbolic language for thinking may be restricted, particularly when they are trying to "think in English," this inability to understand and synthesize the information leads to a decrease in the development of abstract reasoning and the thought processes necessary for problem solving (Tinajero and Dunlap, 1985).

Even when students have fairly high reading skills, they may have difficulties with word problems because of the differences between general prose and math problems. Vos (1979) has researched the language of mathematics extensively and found the following differences:

1. Word problems are more compact and conceptually denser than ordinary prose.
2. The writing style found in word problems is usually different from that used in other types of prose.
3. Word problems often lack the relatively rich context clues typical of other prose.
4. Words with familiar meanings often have different meanings in word problems.
5. Passages in prose usually possess continuity of subject and idea from sentence to sentence and paragraph to paragraph, but there is often little continuity among math word problems in any given set.
6. Reading patterns effective for ordinary prose are usually not as effective for word problems in math. (cited in Ferguson and Fairburn 1985: 505)

The fourth aspect can create considerable difficulty for students unfamiliar with the differences between general and technical vocabularies. The general vocabulary, that is used in everyday conversation on common topics, is usually the one which most students are familiar with. The technical vocabulary is more specific to

mathematics. It is this vocabulary that sometimes requires explicit instruction because many words occur in both the general and the technical vocabularies with different meaning in each.

> These multiple meanings cause children difficulties in reading mathematical material. Because of its more common usage, the general meaning will flash into the child's mind before the technical meaning. Thus the child must overcome the interference caused by the general meaning. (Dunlap and McKnight, 1978: 187)

For example, the words "base", "point", "right", "set", and "imaginary" have meanings in the technical sense which are not immediately recognizable as related to the meanings in the common usage. Of course, this problem may be experienced by students regardless of their language background, but for LEP students it may be an additional burden.

From this, it is apparent that word problems require a translation from"natural" into mathematical language. Added to this is a second translation required from the technical to the symbolic language of numbers. Dunlap and McKnight (1978) identify this three level translation as one of the major problems for not just LEP students, but all students of math.

Vocabulary is not the only linguistic element which presents problems for LEP students. Mathematical syntax is equally challenging, with its abundance of comparative structures, numbers used as nouns, prepositions, passive voice, and lack of one-to-one correspondence between mathematical symbols and the words they represent (Dale and Cuevas, 1987). Dale and Cuevas also document that students tend to duplicate the surface syntax of the problem statements in their numerical restatements. For example, students often incorrectly translated the sentence 'The number a is five less than the number b" as, a = 5 - b, when the correct translation should be: a = b - 5 (1987: 16). Misinterpreting the syntax of the word problem will present major difficulties for the LEP student. It is vital that their reading processes lead to correct interpretations.

Dunlap and McKnight break down the process of reading word problems into eight steps:

1. perceive written words;
2. decode these words using context, phonic, or structural analysis skills;
3. integrate the general (everyday) definitions of each decoded word to arrive at a general message being conveyed in the word problem through semantic and syntactic elements;

4. translate this general message into the technical (mathematical) message being conveyed;
5. encode the technical message into symbolic vocabulary and sentences;
6. perform mathematical calculation on these symbols;
7. decode mathematical calculations and translate to technical vocabulary;
8. encode the technical meaning in terms of response to the technical message, i.e., translate to general vocabulary. (1978: 185)

Clearly, the reading process is not only linguistically complex but also cognitively demanding, and a linguistic problem at any step may produce an incorrect result.

From a slightly different perspective, Reutzel (1983) outlines three major blocks to successfully reading word problems, which he labels "textbase unfamiliarity," "understanding and experience," and "process."

Textbase unfamiliarity refers to the organizational style of word problems as compared with the narratives which the child is accustomed to reading. In his analysis, Reutzel claims that the usual organization in a narrative story or paragraph begins with a general topic and fills out with details, forming a triangle resting on its base. On the other hand, word problems begin with the broad base of details and important facts, narrowing to the thesis or topic sentence, thus creating a triangle balancing on its point. Because the organizational structure does not conform to the child's expectations, there is an added dimension of difficulty.

Understanding and experience is generally lacking in children's reading of word problems. The problems commonly refer to aspects and elements of life completely outside the child's realm of experience. Without an understanding of the context, the problem is even more difficult, if not impossible. For instance, some problems discuss financial investments and interest rates in an effort to give the students practice working with percentages. However, for many if not most children, dividends and interest rates are completely unfamiliar concepts.

Process, in Reutzel's terms, refers to the lack of schema taught for solving word problems. In other words, the child has no heuristic devices to synthesize the given information with cognitive structures. This lack prevents effective integration, sequencing of data, and discrimination of relevant from non-relevant facts, all of which are crucial for successful problem solving.

Many of the difficulties in attempting to solve word problems are certainly encountered by all students regardless of their language background. However, when considering that LEP students have not only the developmental skills to master but also the additional burden of linguistic difficulties, it might seem almost an

insurmountable task to teach LEP students the requisite skills. However, there are some very promising solutions.

Perhaps the ideal solution would be to teach students how to solve word problems in their native languages. This would develop the L1 to a high level of abstraction. Cummins' (1981) model of Common Underlying Proficiency posits that the skills would then transfer to the L2. Tinajero and Dunlap strongly support bilingual education so that the L1 may be used as a "vehicle for developing a conceptual base for solving story problems in both [the native language] and English" (1985: 321).

Unfortunately, for many programs, bilingual instruction for all language minorities is simply not feasible. Therefore, it is necessary to look deeper for solutions that will work in all situations.

In this case, perhaps the most important aspect of mathematics teaching is to build upon the students' previous knowledge. Most students have had experience with mathematics in their native countries. Therefore, it is unnecessary to teach as if the students do not have basic mathematical skills (unless assessment proves this to be true). Of course, skills assessment needs to be done for all students at all levels to ensure accurate placement. Once this is accomplished, the teacher can begin at the appropriate level of instruction.

Building on previous knowledge involves not only knowing the level of the student's output, but also familiarity with how mathematical reasoning is processed. The focus of solving word problems in particular should be on the process of coming to an answer, rather than merely acknowledgement of the correctness of the response. This can be accomplished by direct question; for example, "How did you solve that?" Since word problems can often be solved in more than one way, asking several students how they came up with their solution will likely bring different responses. This will enable other learners to expand their skills. By accepting and understanding different processes, the students' breadth of cognitive flexibility will be increased.

Emphasizing the process over the product has other benefits as well. One of the primary advantages is that it can prevent students from using faulty logic and reasoning in the early stages, thus preventing some possible problems from emerging at more advanced levels.

An added benefit of process-orientation is that it allows for and encourages greater language flexibility. Rather than merely stating their answers, students are required to verbalize their thought processes. This can be a highly effective language teaching tool because their thoughts will be focused on the content of the message rather than the structure.

Of course, the orientation of the class depends upon whether the content or the language is to be the focus, since different emphases require different perspectives and types of activities. If the goal is primarily to develop mathematical concepts, there might be a strong orientation toward problem solving and skills. Language is then viewed as a tool for understanding mathematics. If, on the other hand, the focus is on building language, math will be viewed as merely a vehicle to facilitate language acquisition.

Ideally, the focus of the class would be on both mathematical skills and language development, and the syllabus and lessons would be designed to promote both at the same time. One solution might be to first focus on developing concepts using similar language structures, and then expanding the language with similar problems. This way both math and language skills are integrated.

In working on word problems with LEP students, there are a number of strategies which might prove beneficial toward both types of skills. Certain language modifications can make the mathematical concepts more accessible. For example, simplifying the syntax of the word problems could be done in the same way that the teacher simplifies her own speech. Shorter sentences, reduced complexity, fewer modifiers and conjunctions will assist the student.

Another strategy is to explicitly teach vocabulary and background information which is often assumed to be known in word problems. Discussing special meanings of words will be helpful for the LEP student. For example, "product" commonly means the result of some process, while in the technical vocabulary it refers specifically to the end result of multiplication. Explicit instruction and modeling of specialized meanings will develop the cognitive and linguistic background necessary for learning new mathematical concepts.

Maintaining a basic set of vocabulary would greatly simplify the language. Students are often presented with a wide range of synonyms which are unnecessary when focusing on mathematical concepts. For example, using "minu,s" "less," and "take away" to signify subtraction will present problems for the student who may not immediately recognize these terms as representing the same concept.

Language modification should include simplifying the syntax and vocabulary as necessary to make the word problem comprehensible, but not "substitut[ing] easier words for the technical vocabulary that students must learn in order to acquire the mathematical concepts being presented" (Dunlap and Tinajero, 1985: 164). Doing so only prevents the student from further cognitive development. While it may seem helpful at the moment, it will only prove to be a hinderance at higher levels.

However, once the students are comfortable with the set of terms used, their vocabularies can be expanded, again through explicit instruction. One idea would be

to hang a poster on the classroom wall identifying words that commonly signal certain functions. For example, Reutzel (1983) identifies the following words as pointing to specific operations:

ADDITION	MULTIPLICATION	SUBTRACTION	DIVISION
plus	product	left	into
add	times	take away	separate
sum	by	minus	part
total		difference	
more		less	
increase		decrease	

However, complete reliance on key words to signal specific operations is not advisable. Key words often work only at basic levels. This would eventually prevent students from growing and advancing their skills. "[The] use of key words short-circuits children's natural tendency of trying to figure out problem situations and it communicates that mathematics is little more than a bunch of unrelated rules that are applied in a mindless manner" (Secada and Carey, 1990: 11). Clearly, key words must be used with caution, if at all.

Even if key words always worked, there are other skills which need to be learned. An important skill is discriminating between relevant and irrelevant information. Word problems often include bits of information which have no effect on the outcome of the problem. Dale and Cuevas explain an activity designed specifically for this purpose, involving several word problems written on cards; some have too much information, some too little, and some just enough. If the problem has enough information, the students solve it; if there is too much, they cross it out; if there is too little, they add enough to make sense and then solve it (1987).

Of course, before reaching this level, the student must be able to identify the question being asked. This is sometimes as complex as working out the solution, since questions can be asked at the beginning, middle, or end of the problem. They can be written in the form of an interrogative, a command, or even a declarative. Since identifying the question is of primary importance, this should be emphasized as a beginning step toward the solution.

Another important strategy for solving word problems is to create an image from the given information. This helps the student conceptualize the data and understand what is being asked. It is often easier to identify the operation for the solution when an image of some sort is actually drawn on the board or paper. Selecting the right process is difficult even if all words, situations, and questions are understood (Kresse, 1984). Thus, the teacher should emphasize the importance of mental imagery by

illustrating problems with pictures, charts, and diagrams, and encouraging the students to do the same.

Along with illustrations, manipulatives and other realia are extremely helpful in learning to solve mathematical problems. Such objects can help bring abstract concepts to a concrete level with which students are more familiar. Of course, the focus needs to be on clarifying scenarios rather than on how to use the manipulatives, as can be the case when dealing with sophisticated devices. For example, for most students, using an abacus would necessitate more instruction for its use than is merited by its value in solving problems.

Another method to make word problems more concrete to the student is to make them highly context bound, using situations and experiences which come from the students' lives. Threadgill-Sowder *et al* (1984) emphasize the importance of a sound context, especially for students with lower reading skills. A particularly effective means of ensuring this is to have students create their own word problems. These are likely to be based on personal experiences and problems. Not only would these problems be more meaningful and understandable for the student, but they would be relevant in a way that would increase the student's motivation, which would in turn increase and accelerate her skills and abilities.

In the beginning stages, the teacher would need to extensively model word problem writing. This could involve a language experience approach in which class members contribute to the scenario and the teacher writes it on the board. After exposure to word problem creation, students could work in small groups to begin to produce their own problems. On this topic, Ferguson and Fairburn claim:

> Story problem solving is encouraged by story problem writing, and this is especially true when the problem situation arises from an activity which personally involves the student. Students can be expected to have greater success at interpreting a story problem if they have had experience writing a similar problem. (1985:506)

Working on writing in small groups would have the added advantage of providing students with more language learning opportunities. Small groups such as are used in cooperative learning techniques seem to help the students improve their abilities, in both the language and content areas. Cooperative groups generally provide a secure environment in which students feel freer to express themselves and share ideas and opinions. Using cooperative groups would also be beneficial in attempting to solve word problems, since each student would be able to support and contribute to the group's understanding of and solution to the problem.

Writing word problems provides an excellent opportunity for both mathematical and language development, as well as instilling an appreciation of cultural diversity.

With students of different backgrounds contributing to the creation of word problems, there is likely to be a vast display of multicultural perspectives, showing that mathematics does not have to be impersonal and sterile. Rather, it can be a tremendous, vital source of learning.

With an estimated 4.5 million Limited English Proficient students in the United States, there is clearly a great need for programming designed for the language minority students which will allow them to advance their cognitive abilities at a rate equal to that of their English proficient peers. This can be accomplished by modifying existing curricula and adapting successful techniques. The teaching and learning of mathematics can be facilitated by certain strategies such that solving word problems, once the bane of LEP students, can become a rich and rewarding experience.

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